ΛΛ-ΞN coupling effects in light hypernuclei

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Abstract. The significance of $\Lambda\Lambda$-ΞN coupling in double-Λ hypernuclei has been studied. The Pauli suppression effect due to this coupling in $^{6}_{\Lambda\Lambda}$He has been found to be 0.43 MeV for the coupling strength of the NSC97e potential. This indicates that the free-space $\Lambda\Lambda$ interaction is stronger by an about 5° phase shift than that deduced from the empirical data of $^{6}_{\Lambda\Lambda}$He without including the Pauli suppression effect. In $^{5}_{\Lambda\Lambda}$He and $^{5}_{\Lambda\Lambda}$H, an attractive term arising from the $\Lambda\Lambda$-ΞN conversion is enhanced by the formation of an alpha-particle in the intermediate Ξ states. According to this enhancement, we have found that the $\Lambda\Lambda$ binding energy ($\Delta B_{\Lambda\Lambda}$) of $^{5}_{\Lambda\Lambda}$He is about 0.27 MeV larger than that of $^{6}_{\Lambda\Lambda}$He for the NSC97e coupling strength. This finding deviates from the general picture that the heavier is the core nucleus, the larger is $\Delta B_{\Lambda\Lambda}$.

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1 Introduction

The significant role of ΛN-ΣN coupling in s-shell single-Λ hypernuclei and in neutron matter with a Λ has been presented by us in previous papers [1,2]. It is also our interest to find out the effect of $\Lambda\Lambda$-ΞN coupling in double-Λ hypernuclei, since the mass difference between the Λ and Ξ channels is only 28 MeV, which is smaller than that of the Λ and Σ channels. Recently, in the KEK E373 emulsion counter hybrid experiment a $^{6}_{\Lambda\Lambda}$He, known as "Nagara" event [3], was observed unambiguously. Filikhin and Gal [4] have carried out Faddeev-Yakubovsky three- and four-body calculations to analyze $^{6}_{\Lambda\Lambda}$He and other double-Λ hypernuclei with various Nijmegen OBE YN potential models. However, we notice that the Pauli suppression effect has not been included in their calculations, which motivates us to investigate this effect on the $^{6}_{\Lambda\Lambda}$He, $^{5}_{\Lambda\Lambda}$He and $^{5}_{\Lambda\Lambda}$H binding energies and on the deduction of the $\Lambda\Lambda$ interaction in free space. In this paper we reveal an appreciable Pauli suppression effect of $\Lambda\Lambda$-ΞN coupling in $^{6}_{\Lambda\Lambda}$He and a significant enhancement effect in $^{5}_{\Lambda\Lambda}$He.

2 Λ-nucleus and ΛΛ potentials

We treat the double-Λ hypernuclei as $\Lambda + \Lambda +$ core nucleus three-body systems, and first prepare the necessary

Λ-nucleus and $\Lambda\Lambda$ potentials. A hyperon-nucleon potential, D2, which essentially solves the overbinding problem in s-shell Λ hypernuclei [1], is used to obtain Λ-nucleus folding potentials by the Brueckner-Hartree-Fock method. They are slightly modified [5] so as to reproduce the experimental binding energies of the respective hypernuclei, $^{4}_{\Lambda}$H, $^{4}_{\Lambda}$He and $^{5}_{\Lambda}$He, and are expressed in the following two-range Gaussian form:

$$V_{\Lambda\text{-nucleus}}(r) = \sum_{i=1}^{2} V_i e^{-(\frac{r}{\mu_i})^2} \qquad (1)$$

with the parameters given in table 1.

A single-channel $\Lambda\Lambda$ interaction in free space is derived from the diagonal and off-diagonal terms of a strangeness

Table 1. Strength and range parameters of s-shell Λ-nucleus potentials.

	V_1 (MeV)	V_2 (MeV)	μ_1 (fm)	μ_2 (fm)	B_Λ (MeV)	B_Λ^{exp} (MeV)
$V_{\Lambda\text{-h}}(0^+)$	58.1	-78.4	1.40	1.72	2.39	2.39 ± 0.03
$V_{\Lambda\text{-h}}(1^+)$	71.0	-81.0	1.40	1.72	1.23	1.24 ± 0.04
$V_{\Lambda\text{-t}}(0^+)$	58.1	-76.8	1.40	1.72	2.05	2.04 ± 0.04
$V_{\Lambda\text{-t}}(1^+)$	71.0	-79.6	1.40	1.72	1.01	1.00 ± 0.04
$V_{\Lambda\text{-}\alpha}(l=0)$	91.0	-95.0	1.30	1.70	3.12	3.12 ± 0.02

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Table 2. Strength and range parameters of Shinmura's potentials, where $V_{\Lambda\Lambda,\,\Xi N} \equiv V_{\Lambda\Lambda,\,\Xi^- p} = V_{\Lambda\Lambda,\,\Xi^0 n}$ is given in charge base, not in isospin base.

	V_1 (MeV)	V_2 (MeV)	μ_1 (fm)	μ_2 (fm)
$V_{\Lambda\Lambda,\,\Lambda\Lambda}$(D)	22912	−384.7	0.35	0.85
$V_{\Lambda\Lambda,\,\Xi N}$(D)		50.9		0.85
$V_{\Lambda\Lambda,\,\Lambda\Lambda}$(97e)	18927	−286.8	0.35	0.85
$V_{\Lambda\Lambda,\,\Xi N}$(97e)		108.6		0.85
$V_{\Lambda\Lambda,\,\Lambda\Lambda}$(F)	14080	−198.6	0.35	0.85
$V_{\Lambda\Lambda,\,\Xi N}$(F)		143.7		0.85

Table 3. Strength parameters of the $\Lambda\Lambda$ potentials in units of MeV.

	V_1	V_2	V_3
$V_{\Lambda\Lambda}^{\mathrm{D}}$	22912	−384.7	−73.0
$V_{\Lambda\Lambda}^{\mathrm{D1}}$	22912	−356.5	−73.0
$V_{\Lambda\Lambda}^{\mathrm{D2}}$	22912	−361.9	−73.0
$V_{\Lambda\Lambda}^{\mathrm{D3}}$	22912	−418.7	−73.0
$V_{\Lambda\Lambda}^{\mathrm{e}}$	18927	−286.8	−171.4
$V_{\Lambda\Lambda}^{\mathrm{e1}}$	18927	−311.2	−171.4
$V_{\Lambda\Lambda}^{\mathrm{e2}}$	18927	−336.4	−171.4
$V_{\Lambda\Lambda}^{\mathrm{e3}}$	18927	−391.3	−171.4
$V_{\Lambda\Lambda}^{\mathrm{F}}$	14080	−198.6	−296.2
$V_{\Lambda\Lambda}^{\mathrm{F1}}$	14080	−246.3	−296.2
$V_{\Lambda\Lambda}^{\mathrm{F2}}$	14080	−291.4	−296.2
$V_{\Lambda\Lambda}^{\mathrm{F3}}$	14080	−343.2	−296.2

$S = -2$ coupled interaction as follows:

$$V_{\Lambda\Lambda}^{\mathrm{sc}} = V_{\Lambda\Lambda,\,\Lambda\Lambda} - V_{\Lambda\Lambda,\,\Xi^- p}\frac{1}{\Delta E}V_{\Xi^- p,\,\Lambda\Lambda}$$
$$-V_{\Lambda\Lambda,\,\Xi^0 n}\frac{1}{\Delta E}V_{\Xi^0 n,\,\Lambda\Lambda}. \tag{2}$$

The diagonal term $V_{\Lambda\Lambda,\,\Lambda\Lambda}$ and the coupling terms $V_{\Lambda\Lambda,\,\Xi N}$ are Shinmura's potentials which are phase shift equivalents to the Nijmegen soft-core NSC97e [6] and the Nijmegen hard-core NHC-D [7] and NHC-F [8] potentials; these are in Gaussian forms with the parameters given in table 2. A hard-core radius parameter $r_c = 0.56\ (0.53)$ fm, which is common to all NN and YN $^1S_0$ states, is used in the NHC-D (F) potential. ΔE of eq. (2) is the operator in intermediate propagation, and the single-channel $\Lambda\Lambda$ interaction becomes a non-local potential. For convenience of practical use, we substitute for it a phase shift equivalent local potential in a closure approximation in which the ΔE is replaced by an averaged value $\overline{\Delta E}$. The resultant local single-channel interaction is given by

$$V_{\Lambda\Lambda}^{\mathrm{sc}}(r) = \sum_{i=1}^{3} V_i e^{-(\frac{r}{\mu_i})^2} \tag{3}$$

with $\mu_1 = 0.35$ fm, $\mu_2 = 0.85$ fm and $\mu_3 = 0.60$ fm.

First, we construct the local single-channel $V_{\Lambda\Lambda}^{\mathrm{sc}}$ by adjusting $\overline{\Delta E} > 0$ to reproduce the scattering parameters of the NSC97e $S = -2$ interaction. We found that $\overline{\Delta E} = 137.6$ MeV reproduces the scattering length, $a_{\Lambda\Lambda} = -0.50$ fm, and the effective range, $r_{\Lambda\Lambda} = 8.41$ fm. This local single-channel $\Lambda\Lambda$ potential is given in table 3 as $V_{\Lambda\Lambda}^{\mathrm{e}}$ and it well reproduces the phase shifts of the non-local potential of eq. (2) in the region of $E_{\mathrm{c.m.}} = 0$–15 MeV. A measure of the $\Lambda\Lambda$ interaction is given by

$$\Delta B_{\Lambda\Lambda}\left(_{\Lambda\Lambda}^{\mathrm{A}}\mathrm{X}\right) = B_{\Lambda\Lambda}\left(_{\Lambda\Lambda}^{\mathrm{A}}\mathrm{X}\right) - 2B_\Lambda\left(_{\Lambda}^{\mathrm{A-1}}\mathrm{X}\right). \tag{4}$$

The $\Lambda\Lambda$ binding energy of $_{\Lambda\Lambda}^{6}$He is found to be $\Delta B_{\Lambda\Lambda} = 0.64$ MeV which is about 0.4 MeV smaller than that of the Nagara-event data [3], $\Delta B_{\Lambda\Lambda} = 1.01 \pm 0.20^{+0.18}_{-0.11}$ MeV, when we employ this single-channel $V_{\Lambda\Lambda}^{\mathrm{e}}$ and $V_{\Lambda\text{-}\alpha}$ of table 1. Because this result is in good agreement with that of Filikhin and Gal [4], one may think that we have

reached the same conclusion that NSC97e is an appropriate model for reproducing the recent experimental value of $\Delta B_{\Lambda\Lambda}(_{\Lambda\Lambda}^{6}\mathrm{He})$. However, since the effect of the $\Lambda\Lambda$-ΞN coupling is already included implicitly in the single-channel $V_{\Lambda\Lambda}^{\mathrm{e}}$, we must consider the Pauli suppression effect in $_{\Lambda\Lambda}^{6}$He, where all the $0s$ states are forbidden for a nucleon converted from the $\Lambda\Lambda$-ΞN coupling. Since the Pauli suppression effect would cause a serious change in our results as will be discussed later, the above conclusion about NSC97e should be altered. Before introducing the Pauli suppression, we modify $V_{\Lambda\Lambda}^{\mathrm{e}}$ by adjusting the long-range part of $V_{\Lambda\Lambda,\,\Lambda\Lambda}$ in eq. (2) to fit the value $\Delta B_{\Lambda\Lambda} = 1.01$ MeV recommended from the Nagara event. We call this fitted $\Lambda\Lambda$ interaction $V_{\Lambda\Lambda}^{\mathrm{e1}}$. It is noticed that this is not a $\Lambda\Lambda$ interaction in free space. Carr, Afnan and Gibson [9] discussed the significance of the Pauli suppression effect in deducing the $\Lambda\Lambda$ interaction in free space from the experimental $\Lambda\Lambda$ binding energies of double-Λ hypernuclei.

The Pauli suppression effect in $_{\Lambda\Lambda}^{6}$He is given by

$$\Delta V_{\mathrm{Pauli}} = V_{\Lambda\Lambda,\,\Xi^- p}\frac{P_\alpha}{\Delta M}V_{\Xi^- p,\,\Lambda\Lambda}$$
$$+V_{\Lambda\Lambda,\,\Xi^0 n}\frac{P_\alpha}{\Delta M}V_{\Xi^0 n,\,\Lambda\Lambda}, \tag{5}$$

where P_α is the projection operator on the $0s$ nucleon states in an α-particle. Here, we also restrict the Ξ states to $0s$ to estimate the minimum effect of the Pauli suppression. Then,

$$\Delta M = \frac{M_{\Xi^0} + M_{\Xi^-}}{2} + \frac{M_{\mathrm{p}} + M_{\mathrm{n}}}{2} - 2M_\Lambda + 2B_\Lambda(_{\Lambda}^{5}\mathrm{He})$$
$$= 32.0\ \mathrm{MeV}, \tag{6}$$

where we neglect the $\Lambda\Lambda$ binding energy $\Delta B_{\Lambda\Lambda}$ since it is nearly cancelled by the binding energy of the Ξ [10]. $\Delta V_{\mathrm{Pauli}}$ is expressed as a two-body non-local potential

Table 4. Calculated $-\Delta B_{\Lambda\Lambda}$ for different $\Lambda\Lambda$ interactions in units of MeV.

	$^{6}_{\Lambda\Lambda}$He	$^{5}_{\Lambda\Lambda}$He	$^{5}_{\Lambda\Lambda}$H
$V^{\mathrm{D}}_{\Lambda\Lambda}$	-1.48	-1.13	-1.07
$V^{\mathrm{D1}}_{\Lambda\Lambda}$	-1.01	-0.76	-0.73
$V^{\mathrm{D2}}_{\Lambda\Lambda} + \Delta V^{\mathrm{D}}_{\mathrm{Pauli}} + \Delta V^{\mathrm{D}}_{\mathrm{alpha}}$	-1.01	-0.86	-0.77
$V^{\mathrm{D2}}_{\Lambda\Lambda}$	-1.10	-0.83	-0.79
$\Delta V^{\mathrm{D}}_{\mathrm{Pauli}}$	0.09	0.05	0.05
$\Delta V^{\mathrm{D}}_{\mathrm{alpha}}$	$-$	-0.08	-0.03
$V^{\mathrm{e}}_{\Lambda\Lambda}$	-0.64	-0.47	-0.45
$V^{\mathrm{e1}}_{\Lambda\Lambda}$	-1.01	-0.76	-0.72
$V^{\mathrm{e2}}_{\Lambda\Lambda} + \Delta V^{\mathrm{e}}_{\mathrm{Pauli}} + \Delta V^{\mathrm{e}}_{\mathrm{alpha}}$	-1.01	-1.28	-0.96
$V^{\mathrm{e2}}_{\Lambda\Lambda}$	-1.44	-1.10	-1.03
$\Delta V^{\mathrm{e}}_{\mathrm{Pauli}}$	0.43	0.23	0.20
$\Delta V^{\mathrm{e}}_{\mathrm{alpha}}$	$-$	-0.41	-0.13
$V^{\mathrm{F}}_{\Lambda\Lambda}$	-0.27	-0.20	-0.20
$V^{\mathrm{F1}}_{\Lambda\Lambda}$	-1.01	-0.74	-0.70
$V^{\mathrm{F2}}_{\Lambda\Lambda} + \Delta V^{\mathrm{F}}_{\mathrm{Pauli}} + \Delta V^{\mathrm{F}}_{\mathrm{alpha}}$	-1.01	-1.92	-1.20
$V^{\mathrm{F2}}_{\Lambda\Lambda}$	-1.89	-1.45	-1.36
$\Delta V^{\mathrm{F}}_{\mathrm{Pauli}}$	0.88	0.47	0.43
$\Delta V^{\mathrm{F}}_{\mathrm{alpha}}$	$-$	-0.94	-0.27

Fig. 1. Λ-Λ phase shifts for different cases of the $\Lambda\Lambda$-ΞN coupling strength. The dot-dashed line is for $V^{\mathrm{e1}}_{\Lambda\Lambda}$, the solid line is for $V^{\mathrm{e2}}_{\Lambda\Lambda}$ and the dashed line is for $V^{\mathrm{F2}}_{\Lambda\Lambda}$.

with

$$P_\alpha = |0s0s\rangle_{\alpha\alpha}\langle 0s0s|, \tag{7}$$

which is given in appendix A. We then define the $V^{\mathrm{e2}}_{\Lambda\Lambda}$ as

$$V^{\mathrm{e2}}_{\Lambda\Lambda} = V^{\mathrm{e1}}_{\Lambda\Lambda} - \Delta V^{\mathrm{e}}_{\mathrm{Pauli}}, \tag{8}$$

where the superscript in the second term means that the NSC97e coupling strength is used in eq. (5). In other words, $V^{\mathrm{e2}}_{\Lambda\Lambda} + \Delta V^{\mathrm{e}}_{\mathrm{Pauli}}$ produces $\Delta B_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}\mathrm{He}) = 1.01\,\mathrm{MeV}$, whereas $V^{\mathrm{e2}}_{\Lambda\Lambda}$ alone gives $1.44\,\mathrm{MeV}$ as shown later. This $V^{\mathrm{e2}}_{\Lambda\Lambda}$ is the single-channel $\Lambda\Lambda$ interaction in free space, which reproduces the Nagara-event data, and is more attractive than the fitted $V^{\mathrm{e1}}_{\Lambda\Lambda}$ and the original $V^{\mathrm{e}}_{\Lambda\Lambda}$. Similarly, the single-channel $\Lambda\Lambda$ interactions $V^{\mathrm{D}}_{\Lambda\Lambda}$, $V^{\mathrm{D1}}_{\Lambda\Lambda}$, $V^{\mathrm{D2}}_{\Lambda\Lambda}$, $V^{\mathrm{F}}_{\Lambda\Lambda}$, $V^{\mathrm{F1}}_{\Lambda\Lambda}$ and $V^{\mathrm{F2}}_{\Lambda\Lambda}$ are constructed from the models NHC-D and NHC-F, respectively. The strength parameters of the various $\Lambda\Lambda$ interactions are given in table 3.

3 Results and discussions

We perform the three-body calculations on the $\Lambda + \Lambda + \alpha$, $\Lambda + \Lambda + \mathrm{h}$ and $\Lambda + \Lambda + \mathrm{t}$ systems, where the total wave function of the systems is expanded in terms of t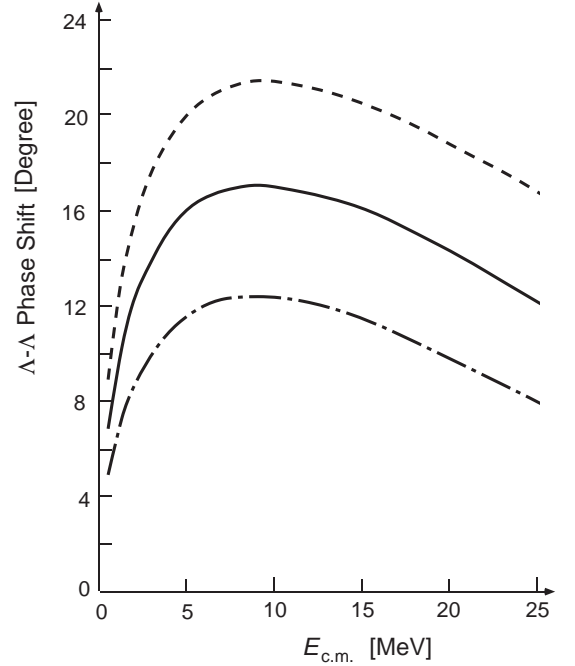he Gaussian wave functions, which are spanned over the three rearrangement channels in Jacobi coordinates [11]. The odd state Λ-α interaction, which is not well determined, is assumed to be zero because its contribution is rather small. A detailed account of our three-body calculation method and its accuracy are discussed in appendix A. The calculated values of the $\Lambda\Lambda$ binding energies with various $\Lambda\Lambda$ potentials are summarized in table 4.

3.1 $\Lambda\Lambda$-ΞN coupling effects in $^{6}_{\Lambda\Lambda}$He

We have derived the single-channel $\Lambda\Lambda$ interactions based on Shinmura's $S = -2$ interactions, which are phase shift equivalents to the NSC97e, NHC-D and NHC-F models. The free-space $\Lambda\Lambda$ interactions, $V^{\mathrm{D2}}_{\Lambda\Lambda}$, $V^{\mathrm{e2}}_{\Lambda\Lambda}$ and $V^{\mathrm{F2}}_{\Lambda\Lambda}$, give $\Delta B_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}\mathrm{He}) = 1.10\,\mathrm{MeV}$, $1.44\,\mathrm{MeV}$ and $1.89\,\mathrm{MeV}$, respectively, and when the Pauli suppression effect is included they reduce to the empirical value, $1.01\,\mathrm{MeV}$. Thus, the Pauli suppression effect due to the $\Lambda\Lambda$-ΞN coupling is $0.09\,\mathrm{MeV}$ for NHC-D, $0.43\,\mathrm{MeV}$ for NSC97e and $0.88\,\mathrm{MeV}$ for NHC-F in $^{6}_{\Lambda\Lambda}$He. The Λ-Λ scattering phase shifts derived from the potentials $V^{\mathrm{e1}}_{\Lambda\Lambda}$, $V^{\mathrm{e2}}_{\Lambda\Lambda}$ and $V^{\mathrm{F2}}_{\Lambda\Lambda}$ are shown in fig. 1. The figure indicates that a $12°$ phase shift at the maximum is obtained for $V^{\mathrm{e1}}_{\Lambda\Lambda}$ if the Nagara-event data is fitted by ignoring the Pauli suppression effect. For $V^{\mathrm{e2}}_{\Lambda\Lambda}$ and $V^{\mathrm{F2}}_{\Lambda\Lambda}$, which are fitted by including the Pauli suppression effect, the Λ-Λ phase shifts attain $17°$ and $22°$, respectively. The Λ-Λ scattering lengths are $a^{\mathrm{e1}}_{\Lambda\Lambda} = -0.73\,\mathrm{fm}$, $a^{\mathrm{e2}}_{\Lambda\Lambda} = -1.03\,\mathrm{fm}$, $a^{\mathrm{F1}}_{\Lambda\Lambda} = -0.67\,\mathrm{fm}$, and $a^{\mathrm{F2}}_{\Lambda\Lambda} = -1.32\,\mathrm{fm}$, respectively. It should be stressed that when we derive the free-space single-channel $\Lambda\Lambda$ interaction from the Nagara-event data, the Pauli effect due to the $\Lambda\Lambda$-ΞN coupling

should be consistently taken into account. In the case of the NSC97e coupling strength, the potential in free space $V_{\Lambda\Lambda}^{e2}$ is much more attractive than $V_{\Lambda\Lambda}^{e1}$ and $V_{\Lambda\Lambda}^{e}$, the latter is equivalent to the original NSC97e $S = -2$ interaction. The situation is more pronounced (restrained) with the NHC-F (NHC-D) interaction, which has a stronger (weaker) $\Lambda\Lambda$-ΞN coupling strength mainly due to the value 3.33 (0.94) of the $f_{\Xi\Lambda K^*}$ baryon-meson coupling constant [12].

Filikhin and Gal [4] showed that the $\Delta B_{\Lambda\Lambda}$ of $^{6}_{\Lambda\Lambda}$He with the NSC97e $\Lambda\Lambda$ interaction is about 0.5 MeV smaller than that of the Nagara event without the Pauli suppression effect. They discussed that the Pauli suppression effect would be completely cancelled out by a higher partial-wave effect. We have checked the contributions of higher partial waves to the binding energy of $^{6}_{\Lambda\Lambda}$He. The contribution due to $\Lambda\Lambda$ interaction in $l = 1$ and higher states is only 0.0005 MeV which is negligibly small. For Λ-α interaction, the corresponding contribution is 0.14 MeV. Our Λ-α interaction for $l = 1$, derived from NSC97e, is less attractive than the one for $l = 0$ given in table 1, and is parametrized with $V_1 = 33.4$ MeV and $V_2 = -39.4$ MeV on the same ranges. Thus, if we treat the NSC97e case in a consistent way, the Pauli suppression effect of 0.43 MeV cannot be cancelled out by the higher partial-wave effect and a net effect of 0.30 MeV remains.

Recently, Nemura *et al.* [13] pointed out that a large rearrangement effect, due to the presence of the Λ-particle, in the internal energy of the α core takes place in $^{5}_{\Lambda}$He. If we take into account the rearrangement effect, we must reproduce with the $\Lambda\Lambda$ interaction the $\Delta B_{\Lambda\Lambda}$ of the Nagara-event data plus ΔB_{rearr} of the rearrangement which is approximately estimated to be 1 MeV by Kohno [14]. The $\Lambda\Lambda$ interactions fitted to the so increased $\Delta B_{\Lambda\Lambda} + \Delta B_{\text{rearr}}(^{6}_{\Lambda\Lambda}\text{He})$ are denoted as $V_{\Lambda\Lambda}^{D3}$, $V_{\Lambda\Lambda}^{e3}$ and $V_{\Lambda\Lambda}^{F3}$, and the respective Pauli suppression effects are 0.12, 0.58 and 1.19 MeV. The maximum value of the $\Lambda\Lambda$ phase shift attains 30° for NSC97e. A more careful investigation on this rearrangement effect shall be done elsewhere.

3.2 Characteristic features of $^{5}_{\Lambda\Lambda}$He and $^{5}_{\Lambda\Lambda}$H

We then investigate the change of the $\Lambda\Lambda$-ΞN coupling effect in $^{5}_{\Lambda\Lambda}$He and $^{5}_{\Lambda\Lambda}$H, where the energy of the Ξ intermediate state is lowered by the formation of an α-particle, which increases the coupling effect [10]. The net change of the coupling effect in five-body hypernuclear systems consists of two effects, ΔV_{Pauli} and ΔV_{alpha}, where the former is the Pauli suppression effect, while the latter is an enhancement effect due to the formation of an alpha-particle in the Ξ channel. These two effects are written as follows:

$$\Delta V_{\text{Pauli}}(^{5}_{\Lambda\Lambda}\text{H}) = V_{\Lambda\Lambda,\,\Xi^0 n}\frac{P_{\text{t}}}{\Delta M_1}V_{\Xi^0 n,\,\Lambda\Lambda}$$
$$+\frac{1}{2}\left(V_{\Lambda\Lambda,\,\Xi^- p}\frac{P_{\text{t}}}{\Delta M_1}V_{\Xi^- p,\,\Lambda\Lambda}\right), \quad (9)$$

where

$$\Delta M_1 = \frac{M_{\Xi^0} + M_{\Xi^-}}{2} + \frac{M_{\text{p}} + M_{\text{n}}}{2}$$
$$-2M_\Lambda + 2B_\Lambda^{\text{av}}(^{4}_{\Lambda}\text{H}) = 28.29\,\text{MeV}, \quad (10)$$

and

$$\Delta V_{\text{alpha}}(^{5}_{\Lambda\Lambda}\text{H}) = \frac{1}{2}\left(V_{\Lambda\Lambda,\,\Xi^- p}\frac{P_{\text{t}}}{\Delta M_1}V_{\Xi^- p,\,\Lambda\Lambda}\right)$$
$$-\frac{1}{2}\left(V_{\Lambda\Lambda,\,\Xi^- p}\frac{P_\alpha}{\Delta M_2}V_{\Xi^- p,\,\Lambda\Lambda}\right) \quad (11)$$

with

$$\Delta M_2 = M_{\Xi^-} + M_\alpha - M_{\text{t}} - 2M_\Lambda + 2B_\Lambda^{\text{av}}(^{4}_{\Lambda}\text{H})$$
$$= 11.04\,\text{MeV}. \quad (12)$$

P_{t} and P_α are the projection operators on the $0s$ states of ^{3}H and ^{4}He, respectively, and B_Λ^{av} is the average of the ground 0^+ and the excited 1^+ levels of $^{4}_{\Lambda}$He or $^{4}_{\Lambda}$H as

$$B_\Lambda^{\text{av}} = \frac{1}{4}B_\Lambda(0^+) + \frac{3}{4}B_\Lambda(1^+). \quad (13)$$

The second term in ΔV_{alpha} corresponds to the coupling effect where the converted proton is in the $0s$ state and forms an alpha-particle with the triton core nucleus. The first term is used to subtract an effect which is already included in the single-channel $\Lambda\Lambda$ interaction. Since the value of ΔM_2 is smaller than that of ΔM_1, ΔV_{alpha} gives an attractive effect.

Similarly, in the case of $^{5}_{\Lambda\Lambda}$He,

$$\Delta V_{\text{Pauli}}(^{5}_{\Lambda\Lambda}\text{He}) = V_{\Lambda\Lambda,\,\Xi^- p}\frac{P_{\text{h}}}{\Delta M_1}V_{\Xi^- p,\,\Lambda\Lambda}$$
$$+\frac{1}{2}\left(V_{\Lambda\Lambda,\,\Xi^0 n}\frac{P_{\text{h}}}{\Delta M_1}V_{\Xi^0 n,\,\Lambda\Lambda}\right) \quad (14)$$

with

$$\Delta M_1 = \frac{M_{\Xi^0} + M_{\Xi^-}}{2} + \frac{M_{\text{p}} + M_{\text{n}}}{2}$$
$$-2M_\Lambda + 2B_\Lambda^{\text{av}}(^{4}_{\Lambda}\text{He}) = 28.83\,\text{MeV}, \quad (15)$$

and

$$\Delta V_{\text{alpha}}(^{5}_{\Lambda\Lambda}\text{He}) = \frac{1}{2}\left(V_{\Lambda\Lambda,\,\Xi^0 n}\frac{P_{\text{h}}}{\Delta M_1}V_{\Xi^0 n,\,\Lambda\Lambda}\right)$$
$$-\frac{1}{2}\left(V_{\Lambda\Lambda,\,\Xi^0 n}\frac{P_\alpha}{\Delta M_2}V_{\Xi^0 n,\,\Lambda\Lambda}\right) \quad (16)$$

with

$$\Delta M_2 = M_{\Xi^0} + M_\alpha - M_{\text{h}} - 2M_\Lambda + 2B_\Lambda^{\text{av}}(^{4}_{\Lambda}\text{He})$$
$$= 5.69\,\text{MeV}, \quad (17)$$

where P_{h} is the projection operator on the $0s$ states of ^{3}He. In $^{5}_{\Lambda\Lambda}$He, ΔV_{alpha} is more attractive than that of $^{5}_{\Lambda\Lambda}$H due to the smaller value of ΔM_2, which can be compared between eqs. (12) and (17).

In our calculations, we use the fitted single-channel $\Lambda\Lambda$ interactions and the averaged values of our Λ-h and Λ-t potentials, which are as follows:

$$V_{\Lambda\text{-h/t}} = \frac{1}{4} V_{\Lambda\text{-h/t}}(0^+) + \frac{3}{4} V_{\Lambda\text{-h/t}}(1^+). \qquad (18)$$

It is to be noted that $\Delta B_{\Lambda\Lambda}$ for the five-body system is defined by

$$\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda} - 2B_\Lambda^{\text{av}} \qquad (19)$$

with B_Λ^{av} of eq. (13) as was reasonably introduced by Filikhin and Gal [4]. The results are displayed in table 4.

The $\Lambda\Lambda$ interaction, $V_{\Lambda\Lambda}^{\text{e1}}$, produces binding energy values, $\Delta B_{\Lambda\Lambda}$, of 0.72 MeV for $^5_{\Lambda\Lambda}$H and 0.76 MeV for $^5_{\Lambda\Lambda}$He as seen in table 4. The corresponding values produced by the free-space $\Lambda\Lambda$ interaction $V_{\Lambda\Lambda}^{\text{e2}}$ are 1.03 MeV and 1.10 MeV, respectively. We then include the coupling effects $\Delta V_{\text{Pauli}}^{\text{e}}$ and $\Delta V_{\text{alpha}}^{\text{e}}$ in our calculations with $V_{\Lambda\Lambda}^{\text{e2}}$. In $^5_{\Lambda\Lambda}$H, the contribution of the Pauli effect is 0.20 MeV and that of the α enhancement effect is -0.13 MeV for the NSC97e coupling strength. Then, the net effect is a weak repulsion, which reduces the value of $\Delta B_{\Lambda\Lambda}$ in $^5_{\Lambda\Lambda}$H by 0.07 MeV.

It is interesting to find that these effects give $\Delta B_{\Lambda\Lambda} = 1.28$ MeV of $^5_{\Lambda\Lambda}$He, which is larger than 1.01 MeV of $^6_{\Lambda\Lambda}$He, since the attractive effect of $\Delta V_{\text{alpha}}^{\text{e}}$ becomes larger than the suppression effect of $\Delta V_{\text{Pauli}}^{\text{e}}$ in $^5_{\Lambda\Lambda}$He. The contributions of $\Delta V_{\text{Pauli}}^{\text{e}}$ and $\Delta V_{\text{alpha}}^{\text{e}}$ in this system are 0.23 MeV and -0.41 MeV. Thus, the value of $\Delta B_{\Lambda\Lambda}$ for $^5_{\Lambda\Lambda}$He is increased by the coupling effects, while they reduce the $\Delta B_{\Lambda\Lambda}$ value of $^5_{\Lambda\Lambda}$H. It should be noted that the value -0.41 MeV of $\Delta V_{\text{alpha}}^{\text{e}}$ is a significant factor not to be neglected in comparison with the empirical value $\Delta B_{\Lambda\Lambda} = 1.01$ MeV of $^6_{\Lambda\Lambda}$He. Filikhin and Gal predicted values of $\Delta B_{\Lambda\Lambda}$ for $^5_{\Lambda\Lambda}$He and $^5_{\Lambda\Lambda}$H, which are smaller than that of $^6_{\Lambda\Lambda}$He, and gave a comment that "the heavier is the core nucleus, the larger is the $\Delta B_{\Lambda\Lambda}$". This contradicts our results, where $\Delta B_{\Lambda\Lambda}$ of $^5_{\Lambda\Lambda}$He is larger than that of $^6_{\Lambda\Lambda}$He.

The values of $\Delta B_{\Lambda\Lambda}$ for the three double-Λ hypernuclei are plotted against the coupling strength in fig. 2. With the value of $\Delta B_{\Lambda\Lambda}$ ($^6_{\Lambda\Lambda}$He) fixed to the experimental value of 1.01 MeV, it can be seen that the values for the five-body double-Λ hypernuclei increase with the coupling strength. Thus, the coupling strength can be sensitively deduced from experimental observations of $^5_{\Lambda\Lambda}$He and $^5_{\Lambda\Lambda}$H.

4 Conclusions

We have derived the single-channel $\Lambda\Lambda$ interactions based on Shinmura's $S = -2$ interactions, which are phase shift equivalents to the Nijmegen soft-core NSC97e, hard-core NHC-D and NHC-F models. The $\Lambda\Lambda$ interaction $V_{\Lambda\Lambda}^{\text{e1}}$ is obtained by fitting the recent $^6_{\Lambda\Lambda}$He experimental data, $\Delta B_{\Lambda\Lambda} = 1.01$ MeV [3], without including the Pauli suppression effect. To obtain the free-space $\Lambda\Lambda$ interaction, however, we have to take into account the Pauli suppression effect in fitting the data, since it is appreciably large.
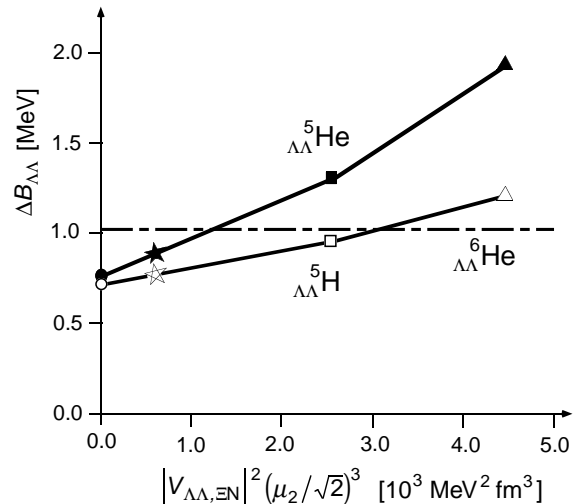
Fig. 2. $\Delta B_{\Lambda\Lambda}$ values against the coupling strength. The circles, stars, squares and triangles represent the cases of no, the NHC-D, the NSC97e and the NHC-F coupling strengths, respectively.

We have found that the Pauli suppression effect in $^6_{\Lambda\Lambda}$He is 0.43 MeV for the NSC97e coupling strength. A Λ-Λ phase shift of 12° at the maximum is produced by the $\Lambda\Lambda$ interaction $V_{\Lambda\Lambda}^{\text{e1}}$, while the free-space $\Lambda\Lambda$ interactions, $V_{\Lambda\Lambda}^{\text{D2}}$, $V_{\Lambda\Lambda}^{\text{e2}}$ and $V_{\Lambda\Lambda}^{\text{F2}}$, give 14°, 17° and 22° maximum values, respectively. Thus, the free-space $\Lambda\Lambda$ interaction of the NSC97e and NHC-F cases is considerably stronger than the interaction which is obtained by fitting the Nagara-event data while neglecting the Pauli suppression effect.

The coupling effects in the five-body systems consist of the Pauli suppression, ΔV_{Pauli}, and an enhancement, ΔV_{alpha}, which arises when a converted nucleon combines with the core nucleus to form an α-particle. These two effects are largely cancelled by each other in $^5_{\Lambda\Lambda}$H, and the resultant effect is a repulsion with 0.07 (0.16) MeV for the NSC97e (NHC-F) coupling strength. In $^5_{\Lambda\Lambda}$He, however, the enhancement effect dominates, and the net coupling effect is not a repulsion but a 0.18 (0.47) MeV attraction for the NSC97e (NHC-F) coupling strength. The behaviour of $\Delta B_{\Lambda\Lambda}$ values against the coupling strength, shown in fig. 2, indicates the significance of the $\Lambda\Lambda$-ΞN coupling effect, and observations of $^5_{\Lambda\Lambda}$He and $^5_{\Lambda\Lambda}$H would be critical for determining the $\Lambda\Lambda$-ΞN coupling strength.

One possible way to produce $^5_{\Lambda\Lambda}$H through the (K^-, K^+) reaction has been discussed by Kumagai-Fuse and Akaishi [15]. They proposed that $^5_{\Lambda\Lambda}$H is almost exclusively formed with a large branching of about 90%, once $^7_{\Xi}$H is populated by the (K^-, K^+) reaction on a ^{7}Li target. This process to produce $^5_{\Lambda\Lambda}$H would be very promising if an intense K^- beam becomes available.

Appendix A.

The total wave function of a three-body system with masses m_1, m_2 and m_3 is expanded in terms of Gaussian basis functions, which are spanned over the three rearrangement channels in the Jacobi coordinates $\vec{r}_c = \vec{r}_i - \vec{r}_j$ and $\vec{R}_c = \vec{r}_k - (m_i \vec{r}_i + m_j \vec{r}_j)/(m_i + m_j)$ as

$$\Psi = \sum_{c=1}^{3} \sum_{i,j}^{N} A_{ij}^c e^{-\left(\frac{r_c}{b_i^c}\right)^2} e^{-\left(\frac{R_c}{b_j^c}\right)^2}, \qquad (A.1)$$

where

$$b_{i+1} = c b_i. \qquad (A.2)$$

Typically, we use $b_1 = 0.2$ fm, $c = 1.4$ and $N = 11$–15. The three-body Hamiltonian is

$$H = -\frac{\hbar^2}{2\mu_c}\nabla_{r_c}^2 - \frac{\hbar^2}{2M_c}\nabla_{R_c}^2 + V_{12} + V_{23} + V_{31}, \qquad (A.3)$$

where μ_c and M_c are the reduced masses corresponding to the Jacobi coordinates. V_{ij}'s are angular-momentum–dependent potentials with Gaussian-form radial parts,

$$\langle \vec{r}' | V | \vec{r} \rangle = \sum_l V_l e^{-\left(\frac{r}{\mu}\right)^2} \frac{\delta(r'-r)}{r^2} \sum_m Y_{lm}^*(\hat{\mathbf{r}}') Y_{lm}(\hat{\mathbf{r}}). \quad (A.4)$$

The matrix elements of the potentials are reduced to the following form:

$$4\pi \left(\frac{\pi}{C}\right)^{\frac{3}{2}} \sum_l (2l+1) V_l \int_0^\infty \mathrm{d}r\, r^2 e^{-Ar^2} i^l j_l(-iBr^2), \quad (A.5)$$

where constants A, B and C are related to the Gaussian-basis parameters of the wave function and range parameters of the potentials. The integral part of the above matrix element is evaluated to be

$$\frac{1}{2}\sqrt{\frac{\pi}{2}} \frac{B^l}{|B|^{2l+1}} \frac{(A - \sqrt{A^2 - B^2})^{l+\frac{1}{2}}}{\sqrt{A^2 - B^2}}. \qquad (A.6)$$

We checked the accuracy of our three-body calculation on the $\Lambda + \Lambda + \alpha$ system with $V_{\Lambda\Lambda}$(97e) and $V_{\Lambda\alpha}$ from Filikhin and Gal [4]. Our calculated value of $B_{\Lambda\Lambda}$, including all partial-wave contributions, is 6.90 MeV, which is in good agreement with 6.90 MeV of Nemura's variational method [16]. When only the s-state interactions are projected out, our calculation gives $B_{\Lambda\Lambda} = 6.70$ MeV, while

that of Filikhin and Gal is 6.82 MeV. Thus, there is a discrepancy between our calculation and theirs; but, in the case of $V_{\Lambda\Lambda} = 0$, our calculated value of $B_{\Lambda\Lambda}$ is 6.27 MeV, which coincides with their value. From our calculation, it is found that the higher partial-wave contribution to the binding energy of $^6_{\Lambda\Lambda}$He is 0.20 MeV when all the interactions are taken to be of s-state. However, when we apply less attractive p-state Λ-α potential to the odd states, the contribution reduces to 0.14 MeV.

The non-local potentials ΔV_{Pauli} and ΔV_{alpha} are included in our calculations by

$$\langle \vec{r}' \vec{R}' | \frac{P}{\Delta M} | \vec{r} \vec{R} \rangle =$$
$$\left(\frac{a}{\pi}\right)^3 e^{-\frac{1}{4}ar'^2} e^{-aR'^2} \frac{1}{\Delta M} e^{-\frac{1}{4}ar^2} e^{-aR^2}, \qquad (A.7)$$

where the harmonic-oscillator strength is taken to be $a = 0.521\,\text{fm}^{-2}$ for ^{4}He and $a = 0.386\,\text{fm}^{-2}$ for ^{3}He and ^{3}H.

References

1. Y. Akaishi, T. Harada, S. Shinmura, Khin Swe Myint, Phys. Rev. Lett. **84**, 3539 (2000).
2. S. Shinmura, Khin Swe Myint, T. Harada, Y. Akaishi, J. Phys. G: Nucl. Part. Phys. **28**, L1 (2002).
3. H. Takahashi *et al.*, Phys. Rev. Lett. **87**, 212502 (2001).
4. I.N. Filikhin , A. Gal, Phys. Rev. C **65**, 041001(R) (2002); Nucl. Phys. A **707**, 491 (2002).
5. Khin Swe Myint, Y. Akaishi, to be published in Prog. Theor. Phys.
6. Th.A. Rijken, V.G.J. Stoks, Y. Yamamoto, Phys. Rev. C **59**, 21 (1999).
7. M.M. Nagels, T.A. Rijken, J.J. de Swart, Phys. Rev. D **15**, 2547 (1977).
8. M.M. Nagels, T.A. Rijken, J.J. de Swart, Phys. Rev. D **20**, 1633 (1979).
9. S.B. Carr, I.R. Afnan, B.F. Gibson, Nucl. Phys. A **625**, 143 (1997).
10. Khin Swe Myint, Y. Akaishi, Prog. Theor. Phys. Suppl. **117**, 251 (1994).
11. M. Kamimura, Phys. Rev. A **38**, 621 (1988).
12. Y. Yamamoto, H. Bando, Prog. Theor. Phys. Suppl. **81**, 9 (1985).
13. H. Nemura, Y. Akaishi, Y. Suzuki, Phys. Rev. Lett. **89**, 142504 (2002).
14. M. Kohno, private communication.
15. I. Kumagai-Fuse, Y. Akaishi, Phys. Rev. C **54**, 24 (1996).
16. H. Nemura, private communication.